SAMPLE

MATH 5A - TEST 1 v2
(CHAPTER 1.4-1.8, 2.1i , 3.4i and ii)

100 points **NAME:** Solutions

YOU MUST SHOW YOUR WORK. PRESENTATION COUNTS, use words to explain the processes.
Phones must be OFF and put away. No graphing calculators allowed. No scratch paper allowed.

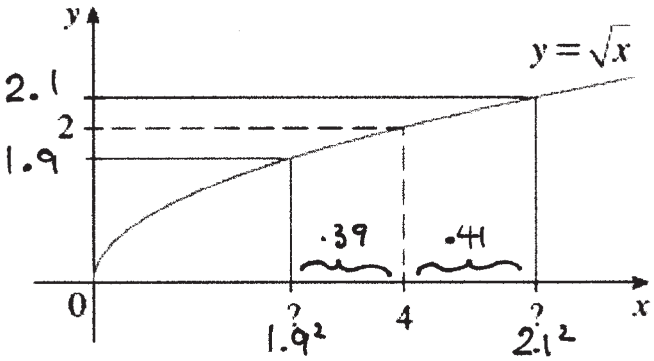
FILL IN THE BLANKS WITH MOST APPROPRIATE ANSWER: (2 points each)

(1) Give either form of the difference quotient definition of $f'(a)$ $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ or $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

(2) True or False: $\lim_{x\to a}\tan(x)=\tan(a)$ for all values of a. No. Not true where $\tan x$ discont. like at $a=\pi/2$

(3) For $f(x)=\sqrt{x}$, find the number delta corresponding to an epsilon of $\varepsilon=0.1$ so that if $0<|x-4|<\delta$ then $|f(x)-2|<\varepsilon$ Use the graph if desired.

(4 points)



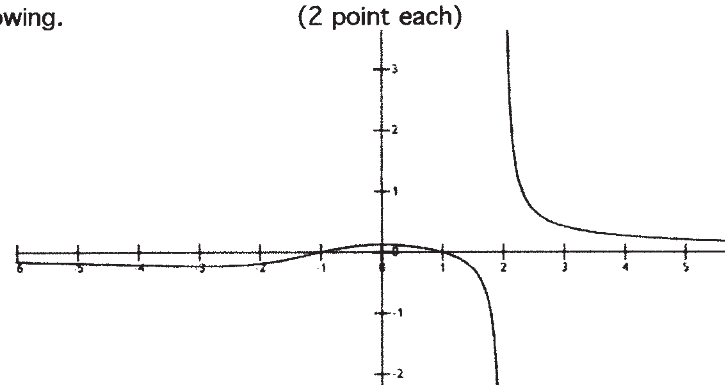
$\delta = 0.39$

(4) Use the given graph of f(x) to find each of the following. (2 point each)
(if the limit is ∞ or $-\infty$ say so.)

a) $\lim_{x\to\infty} f(x)=$ 0

b) $\lim_{x\to 2^-} f(x)=$ $-\infty$

b) $\lim_{x\to 2^+} f(x)=$ ∞



(5) Suppose you are trying to prove $\lim_{x\to 1}(3x-7)=-4$ Given $\varepsilon>0$ what value must δ be in order to satisfy the definition of limit? (No need to show formal proof) (4 points)

$0<|x-a|<\delta \Rightarrow |f(x)-L|<\varepsilon$

$|x-1|<\delta \Rightarrow |3x-7-{-4}|<\varepsilon$

$|3x-3|<\varepsilon$

$|x-1|<\frac{\varepsilon}{3}$

$\delta=\frac{\varepsilon}{3}$

(6) (a) Give the formal/rigorous definition for $\lim_{x \to a^+} f(x) = L$ (9 points)

For every $\varepsilon > 0$ there is a $\delta > 0$ such that if $a < x < a + \delta$ then $|f(x) - L| < \varepsilon$

(b) Give the formal/ rigorous definition for $\lim_{x \to a} f(x) = -\infty$

For every $M < 0$ there is a $\delta > 0$ such that if $0 < |x - a| < \delta$ then $f(x) < M$

(c) Give the formal/ rigorous definition for $\lim_{x \to \infty} f(x) = \infty$

For every $M > 0$ there is an $N > 0$ such that if $x > N$ then $f(x) > M$

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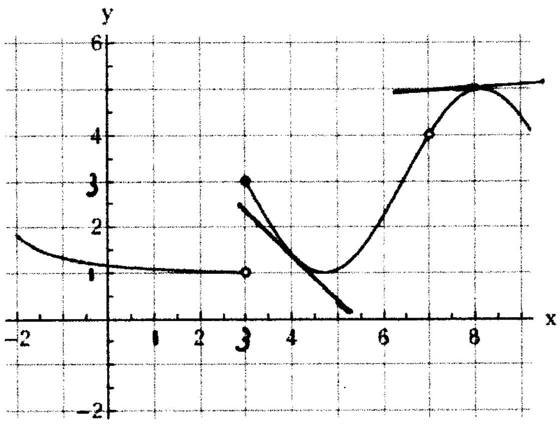
(7) Given the graph of f(x) below, state the value of the following limits if they exist

(1 points each)



(a) $\lim_{x \to 3^-} f(x) =$ 1

(b) $\lim_{x \to 3^+} f(x) =$ 3

(c) $\lim_{x \to 3} f(x) =$ DNE

(d) f(3)= 3

(e) $\lim_{x \to 7} f(x) =$ 4

(f) Approximate $f'(4) \approx$ -1

(g) Approximate $f'(8) \approx$ 0

(h) find c so that f(c) = 5 8

(8) Evaluate the following limits if they exist (if the limit is ∞ or -∞ say so.). No proof or detailed steps necessary, but do show work. (4 points each)

(a) $\lim_{x\to -2} \sqrt[3]{5+x^5} =$ -3

$\sqrt[3]{5+(-2)^5} = \sqrt[3]{-27}$

(b) $\lim_{x\to 3} \frac{x-3}{x^2-9} =$ $\frac{1}{6}$

$= \lim_{x\to 3} \frac{x-3}{(x-3)(x+3)} = \lim_{x\to 3} \frac{1}{x+3} = \frac{1}{6}$

(c) $\lim_{x\to 16} \frac{16-x}{\sqrt{x}-4} =$ -8

$\frac{0}{0}$

$\lim_{x\to 16} \frac{16-x}{\sqrt{x}-4} \cdot \frac{\sqrt{x}+4}{\sqrt{x}+4} = \lim_{x\to 16} \frac{(16-x)(\sqrt{x}+4)}{x-16}$

$= \lim_{x\to 16} (-1)(\sqrt{x}+4) = -8$

(d) $\lim_{x\to 4^-} \frac{x-5}{x(x-4)} =$ ∞

$\frac{-1}{4\cdot 0}$ ↑ neg

$\frac{-}{-} \to +$

(e) If $f(x)=\begin{cases} 3-x^2 & \text{if } x<2 \\ x^4-1 & \text{if } x\geq 2 \end{cases}$ find

$\lim_{x\to 2^+} f(x) =$ 15

$\lim_{x\to 2^+} x^4-1$

$\lim_{x\to 2^-} f(x) =$ -1

$\lim_{x\to 2^-}(3-x^2)$

$\lim_{x\to 2} f(x) =$ dne

(f) $\lim_{x\to -\infty} \frac{4x}{\sqrt{x^2-7}} =$ -4 (Show details)

Since $\sqrt{\frac{1}{x^2}} = \frac{1}{|x|} = \frac{1}{-x}$ when $x<0$ (here $x\to -\infty$)

$\lim_{x\to -\infty} \frac{4x}{\sqrt{x^2-7}} \cdot \frac{\sqrt{1/x^2}}{\sqrt{1/x^2}} = \lim_{x\to -\infty} \frac{4x\left(\frac{1}{-x}\right)}{\sqrt{1-7/x^2}} = \lim_{x\to -\infty} \frac{-4}{\sqrt{1-7/x^2}} = -4$

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(9) Prove that there is at least one solution to the equation cosx = x (4 points)

Hint: If you are going to use a theorem, name the theorem and verify any hypotheses are satisfied.

Proof: Consider $f(x) = \cos x - x$. Showing $f(x)$ has a zero is equivalent to showing $\cos x = x$ has a solution.

Since $f(x)$ is continuous, with $f(0)=1>0$ and $f(\frac{\pi}{2}) = -\frac{\pi}{2} < 0$, by the intermediate value theorem the exists a value $c \in (0/\pi/2)$ such that $f(c)=0$.

∴ There is a solution to $\cos x = x$ ∎

(10) For what values of x are the following functions continuous? Show work. (4 points each)

a) $f(x) = \dfrac{2x+3}{\sin x - 1}$ b) $f(x) = \sqrt{x^2 - x - 6}$ c) $f(x) = \begin{cases} 5x+2 & \text{if } x > 0 \\ \sqrt{4-x} & \text{if } x \le 0 \end{cases}$

Continuous on domain, Just find domain.

a) denom $\neq 0 \Rightarrow$

$\sin x - 1 \neq 0$

$\sin x \neq 1$

$x \neq \frac{\pi}{2} + 2\pi k$

$f(x)$ is conts for all x except $\frac{\pi}{2} + 2\pi k$, k integer.

b) radicand ≥ 0

$x^2 - x - 6 \geq 0$

$(x-3)(x+2) \geq 0$

$+ \quad -2 \quad - \quad 3 \quad +$ sign chart

$f(x)$ conts on $(-\infty, -2] \cup [3, \infty)$

c) Conts for all x except possibly $x=0$. Check $x=0$.

$\lim_{x\to 0^+} f(x) = 2 = \lim_{x \to 0^-} f(x)$.

Since $\lim_{x\to 0} f(x) = f(0)$, f conts at $x=0$ also.

$\therefore f$ conts. on $(-\infty, \infty)$

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(11) The displacement (in meters) of an object moving in a straight line is given by $s = t^2 - 3t$, where t is measured in seconds.

(a) find the average velocity over the time period [4,5] (2 pts.)

(b) using methods discussed in this class, find the instantaneous velocity when t=4. (7 pts)

Answer should have the appropriate units.

units!

a) $V_{ave}[4,5] = \dfrac{s(5) - s(4)}{5-4} = \dfrac{10-4}{1} = 6$ m/sec

b) Instantaneous velocity at $t=4$ $= s'(4) = \lim_{t\to 4} \dfrac{s(t) - s(4)}{t-4} = \lim_{t \to 4} \dfrac{t^2 - 3t - 4}{t-4} = \lim_{t\to 4} (t+1) = 5$ m/sec

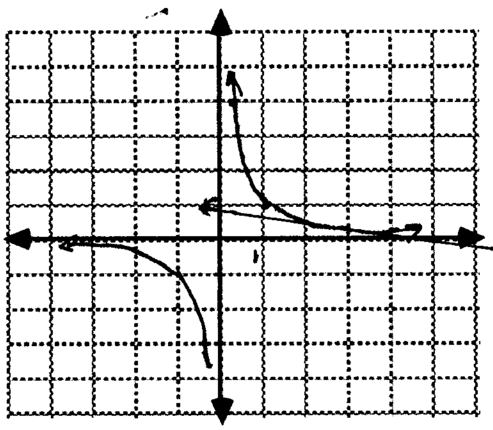
(12) Using methods discussed in class,

a) Use an appropriate form of the definition of the derivative to compute $f'(a)$ if $f(x) = \frac{1}{x}$. (6 pts)

b) Use the results of part (a) to find the slope of the tangent line at x = -1, 1/2, and 3. (3 pts)

c) Sketch a graph of f(x) and the tangent line at x=3. Based on your graph, Is your answer in part (b) reasonable? Explain. (4 pts)

d) Find the equation of the tangent line at x=3. (3 pts)



Tangent at x=3 should have negative slope close to 0

a) $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{\frac{1}{x}-\frac{1}{a}}{x-a} \cdot \frac{ax}{ax} = \lim_{x\to a} \frac{a-x}{ax(x-a)} = \lim_{x\to a} \frac{-1}{ax} = -\frac{1}{a^2}$

$f'(a) = \frac{-1}{a^2}$ (note, not $-\frac{1}{x^2}$)

b) $f'(-1) = -1$

$f'(1/2) = -4$

$f'(3) = -1/9$ ← Reasonable since tangent line decreasing and relatively flat.

(c) Explanation

d) Point $(3, f(3)) = (3, \frac{1}{3})$

Slope $f'(3) = -\frac{1}{9}$

Tangent line is $y - \frac{1}{3} = -\frac{1}{9}(x-3)$

or

$x + 9y = 6$